# Energy System Modelling

SS 2018, Karlsruhe Institute of Technology
Institute of Automation and Applied Informatics

## Tutorial II: Network Theory and Power Flow

Will be worked on in the exercise session on Friday, 13 July 2018.

### Problem II.1 (network theory basics).

Consider the simple network shown ins Figure 2. Calculate in Python or by hand:

(a) Compile the *nodes list* and the *edge list*.

**Remark:** While graph-theoretically both lists are unordered sets, let's agree on an ordering now which can serve as basis for the matrices in the following exercises: we sort everything in ascending numerical order, i.e. node 1 before node 2 and edge (1,2) before (1,4) before (2,3).

(b) Determine the *order* and the *size* of the network.

(c) Compute the *adjacency matrix* $A$ and check that it is symmetric.

(d) Find the *degree* $k_n$ of each node $n$ and compute the *average degree* of the network.

(e) Determine the *incidence matrix* $K$ by assuming the links are always directed from smaller-numbered node to larger-numbered node, i.e. from node 2 to node 3, instead of from 3 to 2.

(f) Compute the *Laplacian* $L$ of the network using $k_n$ and $A$. Remember that the Laplacian can also be computed as $L = KK^T$ and check that the two definitions agree.

(g) Find the *diameter* of the network by looking at Figure 2.

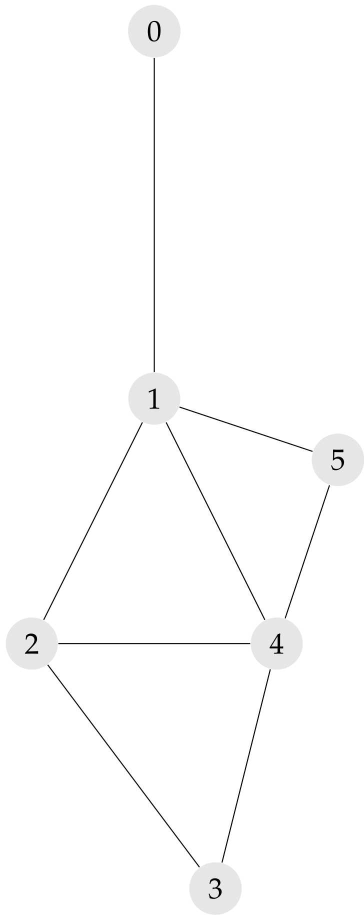


**Figure 1:** Simple Network

**Problem II.2 (linear power flow).**

If you map the nodes to countries like `0=DK, 1=DE, 2=CH, 3=IT, 4=AT,5=CZ` the network in Figure 2 represents a small part of the European electricity network (albeit very simplified). On the course homepage[1], you can find the *power imbalance* time series for the six countries for January 2017 in hourly MW in the file `imbalance.csv`. They have been derived from physical flows as published by ENTSO-E.[2]

The linear power flow is given by

$$p_i = \sum_j \tilde{L}_{i,j}\theta_j \qquad \text{and} \qquad f_l = \frac{1}{x_l}\sum_i K_{i,l}\theta_i, \qquad \text{where} \qquad \tilde{L}_{i,j} = \sum_l = K_{i,l}\frac{1}{x_l}K_{j,l} \tag{1}$$

is the weighted Laplacian. For simplicity, we assume identity reactance on all links $x_l = 1$.

(a) Compute the *voltage angles* $\theta_j$ and *flows* $f_l$ for the first hour in the dataset with the convention of $\theta_0 = 0$; i.e. the slack bus is at node 0.

**Remark:** Linear equation systems are solved efficiently using `numpy.linalg.solve`.

(b) Determine the average flow on each link for 01-2017 and draw it as a directed network.

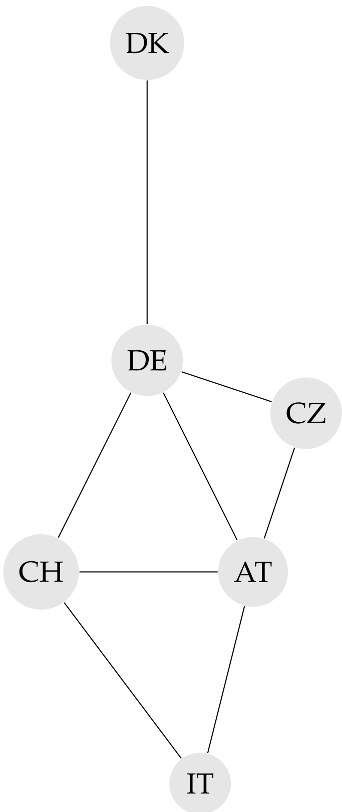


**Figure 2:** Simple Network

[1] https://nworbmot.org/courses/complex_renewable_energy_networks/
[2] https://transparency.entsoe.eu/transmission-domain/physicalFlow/show