Energy System Modelling

SS 2018, Karlsruhe Institute of Technology
Institute of Automation and Applied Informatics

TUTORIAL IV: ELECTRICITY MARKETS

Will be worked on in the exercise session on Tuesday, 17 July 2018.

SOLUTION IV.1 (SHADOW PRICES OF LIMITS ON CONSUMPTION).

Suppose that the utility for the electricity consumption of an industrial company is given by

$$U(q) = 70q - 3q^2 [€/h] \quad , \quad q_{min} = 2 \leq q \leq q_{max} = 10,$$

where q is the demand in MW and q_{min}, q_{max} are the minimum and maximum demand.

Assume that the company is maximising its net surplus for a given electricity price π, i.e. it maximises $\max_q [U(q) - \pi q]$.

(a) If the price is $\pi = 5$ €/MWh, what is the optimal demand q^*? What is the value of the KKT multiplier μ_{max} for the constraint $q \leq q_{max} = 10$ at this optimal solution? What is the value of μ_{min} for $q \geq q_{min} = 2$?

We convert the exercise to an optimisation problem with objective

$$\max_q U(q) - \pi q \tag{1}$$

with constraints

$$q \leq q_{max} \qquad \leftrightarrow \qquad \mu_{max} \tag{2}$$

$$-q \leq -q_{min} \qquad \leftrightarrow \qquad \mu_{min} \tag{3}$$

From stationarity we get:

$$0 = \frac{\partial}{\partial q}(U(q) - \pi q) - \mu_{max}\frac{\partial}{\partial q}(q - q_{max}) - \mu_{min}\frac{\partial}{\partial q}(-q + q_{min}) \tag{4}$$

$$= U'(q) - \pi - \mu_{max} + \mu_{min} \tag{5}$$

The marginal utility curve is $U'(q) = 70 - 6q$ [€/MWh]. At $\pi = 5$, the demand would be determined by $5 = 70 - 6q$, i.e. $q = 65/6 = 10.8333$, which is above the consumption limit $q_{max} = 10$. Therefore the optimal demand is $q^* = 10$, the upper limit is binding $\mu_{max} \geq 0$ and the lower limit is non-binding $\mu_{min} = 0$.

To determine the value of μ_{max} we use (5) to get $\mu_{max} = U'(q^*) - \pi = U'(10) - 5 = 5$.

(b) Suppose now the electricity price is $\pi = 60$ €/MWh. What are the optimal demand q^*, μ_{max} and μ_{min} now?

At $\pi = 60$, the demand would be determined by $60 = 70 - 6q$, i.e. $q = 10/6 = 1.667$, which is below the consumption limit $q_{min} = 2$. Therefore the optimal demand is $q^* = 2$, the upper limit is non-binding $\mu_{max} = 0$ and the lower limit is binding $\mu_{min} \geq 0$.

To determine the value of μ_{min} we use (5) to get $\mu_{min} = \pi - U'(q^*) = 60 - U'(2) = 2$.

Problem IV.2 (Economic dispatch in a single bidding zone).

Consider an electricity market with two generator types, one with variable cost $c = 20$ €/MWh, capacity $K = 300$ MW and a dispatch rate of Q_1 [MW] and another with variable cost $c = 50$ €/MWh, capacity $K = 400$ MW and a dispatch rate of Q_2 [MW]. The demand has utility function $U(Q) = 8000Q - 5Q^2$ [€/h] for a consumption rate of Q [MW].

(a) What are the objective function and constraints required for an optimisation problem to maximise short-run social welfare in this market?

The optimisation problem has objective function:

$$\max_{Q,Q_1,Q_2} \left[U(Q) - C_1(Q_1) - C_2(Q_2)\right] = \max_{Q,Q_1,Q_2} \left[8000Q - 5Q^2 - c_2Q_1 - c_2Q_2\right]$$

with constraints:

$$\begin{aligned} Q - Q_1 - Q_2 = 0 &\leftrightarrow \lambda \\ Q_1 \leq K_1 &\leftrightarrow \bar{\mu}_1 \\ Q_2 \leq K_2 &\leftrightarrow \bar{\mu}_2 \\ -Q_1 \leq 0 &\leftrightarrow \underline{\mu}_1 \\ -Q_2 \leq 0 &\leftrightarrow \underline{\mu}_2 \end{aligned}$$

(b) Write down the Karush-Kuhn-Tucker (KKT) conditions for this problem.

Stationarity gives for Q:

$$\frac{\partial U}{\partial Q} - \lambda = 8000 - 10Q - \lambda = 0$$

Stationarity for Q_1 gives:

$$-\frac{\partial C_1}{\partial Q_1} + \lambda - \mu_1 = -c_1 + \lambda - \bar{\mu}_1 + \underline{\mu}_1 = 0$$

Stationarity for Q_2 gives:

$$-\frac{\partial C_2}{\partial Q_2} + \lambda - \mu_2 = -c_2 + \lambda - \bar{\mu}_2 + \underline{\mu}_2 = 0$$

Primal feasibility is just the constraints above. Dual feasibility is $\bar{\mu}_i, \underline{\mu}_i \geq 0$ and complementary slackness is $\bar{\mu}_i(Q_i - K) = 0$ and $\underline{\mu}_i Q_i = 0$ for $i = 1, 2$.

(c) Determine the optimal rate of production of the generators and the value of all KKT multipliers. What is the interpretation of the respective KKT multipliers?

The marginal utility at the full output of the generators, $K_1 + K_2 = 700$ MW is $U'(700) = 8000 - 10 \cdot 700 = 1000$ €/MWh, which is higher than the costs c_i, so we'll find optimal rates $Q_1^* = K_1$ and $Q_2^* = K_2$ and $Q^* = K_1 + K_2$. This means $\lambda = U'(K_1 + K_2) = 1000$ €/MWh, which is the market price. Because the lower constraints on the generator output are not binding, from complementary slackness we have $\underline{\mu}_i = 0$. The upper constraints are binding, so $\bar{\mu}_i \geq 0$. From stationarity $\bar{\mu}_i = \lambda - c_i$, which is the increase in social welfare if Generator i could increase its capacity by a marginal amount.

PROBLEM IV.3 (EFFICIENT DISPATCH IN A TWO-BUS POWER SYSTEM).

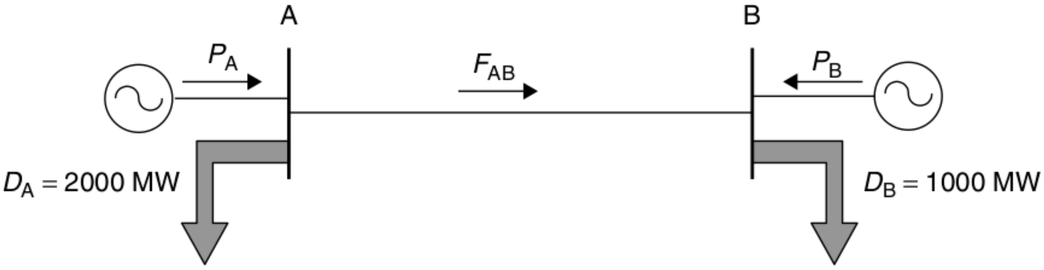

Figure 1: A simple two-bus power system.

Consider the two-bus power system shown in Figure 1, where the two nodes represent two markets, each with different total demand, and one generator at each node. At node A the demand is $D_A = 2000\text{MW}$, whereas at node B the demand is $D_B = 1000\text{MW}$. Furthermore, there is a transmission line with a capacity denoted by F_{AB}. The marginal cost of production of the generators connected to buses A and B are given respectively by the following expressions:

$$
\begin{aligned}
MC_A &= 20 + 0.03P_A \qquad && \text{€}\,/\text{MW h}\\
MC_B &= 15 + 0.02P_B \qquad && \text{€}\,/\text{MW h}
\end{aligned}
$$

Assume that the demand D_* is constant and insensitive to price, that energy is sold at its marginal cost of production and that there are no limits on the output of the generators.

(a) Calculate the price of electricity at each bus, the production of each generator, the flow on the line, and the value of any KKT multipliers for the following cases:

Use the following nomenclature: price λ_i, production Q_i^S, flow F.

(i) The line between buses A and B is disconnected.

$\lambda_A = 80$ €/MWh, $\lambda_B = 35$ €/MWh,
$Q_A^S = 2000$ MW, $Q_B^S = 1000$ MW, $F = 0$

(ii) The line between buses A and B is in service and has an unlimited capacity.

$\lambda_A = 53$ €/MWh, $\lambda_B = 53$ €/MWh,
$Q_A^S = 1100$ MW, $Q_B^S = 1900$ MW, $F = -900$ MW

(iii) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator B is 1500 MW.

$\lambda_A = 65$ €/MWh, $\lambda_B = 65$ €/MWh,
$Q_A^S = 1500$ MW, $Q_B^S = 1500$ MW, $F = -500$ MW

(iv) The line between buses A and B is in service and has an unlimited capacity, but the maximum output of Generator A is 900 MW. The output of Generator B is unlimited.

$\lambda_A = 57$ €/MWh, $\lambda_B = 57$ €/MWh,
$Q_A^S = 900$ MW, $Q_B^S = 2100$ MW, $F = -1100$ MW

(v) The line between buses A and B is in service but its capacity is limited to 600 MW. The output of the generators is unlimited.

$\lambda_A = 62$ €/MWh, $\lambda_B = 47$ €/MWh,
$Q_A^S = 1400$ MW, $Q_B^S = 1600$ MW, $F = -600$ MW

(b) Calculate the generator revenues, generator profits, consumer payments and consumer net surplus for all the cases considered in the above problem. Who benefits from the line connecting these two buses?

Generator revenues R_i, generator costs C_i, generator profits P_i, consumer payments E_i. Find the generator profits by substracting the costs from the revenue. Costs are given by integrating the marginal cost, i.e. $C_A = 20Q + 0.015Q^2$ and $C_B = 15Q + 0.01Q^2$. The generator at B and the consumers at A benefit from the line (price increases at B, decreases at B).

Case	i	ii	iii	iv	v
E_A (€)	160000	106000	130000	114000	124000
E_B (€)	35000	53000	65000	57000	47000
R_A (€)	160000	58300	97500	51300	86800
R_B (€)	35000	100700	97500	119700	75200
C_A (€)	100000	40150	63750	30150	57400
C_B (€)	25000	64600	45000	75600	49600
P_A (€)	60000	18150	33750	21150	29400
P_B (€)	10000	36100	52500	44100	25600

(c) Calculate the congestion surplus for case (v). For what values of the flow on the line between buses A and B is the congestion surplus equal to zero?

Congestion surplus is 9000 €:

$$(E_A + E_B) - (R_A + R_B) = |F| \times (\lambda_A - \lambda_B)$$

Congestion surplus is equal to zero when the flow $F = 0$, or when it is equal to the unconstrained value $F = -900$ MW (then $\lambda_A = \lambda_B$).

PROBLEM IV.4 (BIDDING IN AFRICA WITH PYPSA).